MATCHED FILTERING OF OPTICAL DYNAMIC SIGNALS

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ABSTRACT

Matched filtering of optical dynamic signals in the form of truncated in time monochromatic radiation is investigated. The expression of transmission function was obtained for optical network in the form of. diffractional optical spectrum analyzer.

It was shown that transmission function of that network is matched with complex spectrum of optical dynamic signals in the form of truncated monochromatic light oscillation, i.e. this spectrum analyzer is matched filter for the signals of this kind

Keywords: optical signal, matched filtering, optical network, transmission function.

INTRODUCTION

Matched filtering of radio-signals is a well-developed concept in radio engineering. However, matched filtering methods are not widely used in optical range. At the present time, as the optical range is used for optical location and telecommunication, the problems of optimal filtering of optical signals in the form of space-time wave packets (dynamics signals) becomes more and more topical.

This paper is devoted to the investigation of matched filtering of dynamic signals in the optical range by an optical four-pole network, similar to optical diffractional spectrum analyzer.

The most important characteristics of a matched filter are its transmission function and pulse response, i.e. a response to the $\delta-$function in time domain. Within the frameworks of the radio-optics approach, the transmission function of an optical network in the form of optical diffractional spectrum analyzer was obtained.

The pulse response of optical network was not discussed here, because this characteristic is very difficult to obtain in the optical range. Furthermore, in the optical range both classical and quantum description of physical phenomena are used, and $\delta-$pulse and hence the pulse response of optical network becomes difficult to interpret under such conditions.

LINEAR OPTICAL NETWORK

The optical network under study is a linear optical system, including a grating, Fourier-lens and output plane. The operation of optical network is described in the Cartesian coordinates x, y, z.

The output plane of the grating is the same as the front focal plane of the Fourier-lens, and the output plane of the optical system is the same as the back focal plane of the Fourier-lens. The grating is an amplitude transparency, which transmission function being expressed by the following relation:

$$\mathrm{T}(x_1) = \mathrm{T}_0 \mathrm{r}(x_1)(1 + m \cos \mathrm{K}_0\, x_1), \qquad (1)$$

where T_0 is a constant.; m is the modulation coefficient; x_1 is one of Cartesian coordinate of optical system in the output plane of the grating; K_0 is the grating's space frequency;

$$\mathrm{r}(x_1) = \begin{cases} 1, |x_1| \le 0{,}5L \\ 0, |x_1| \ge 0{,}5L, \end{cases} \qquad (2)$$

where L is the aperture size of the optical system and grating.

Three diffraction orders (maximums) are formed in the back focal plane: a zero order, and two first (principal) orders. The positions of principal orders are determined by the parameters of optical system, the frequency of incident light and, also by the space frequency harmonic function written into the grating.

The Fourier-lens and two layers of free space compose an optical coherent Fourier-processor, performing the spatial Fourier transformation [Ref. 1], the kernel of this transformation being

$$\mathrm{Q}(x_1, x_2, \omega') = \exp(i\omega' x_1, x_2 / cF), \qquad (3)$$

where x_2 is one of Cartesian coordinates in the output plane of the optical system; ω' is the circular frequency of light wave falling onto the grating input plane; c is the speed of light; F is the focal distance of the lens; $\omega' x_2 / cF = \omega_x$ is the spatial frequency.

TRANSMISSION FUNCTION OF OPTICAL NETWORK

A transmission function of a linear optical network is defined under the following conditions: the input of this network is a plane monochromatic light wave, falling normally onto grating, and the output is light oscillations at the place where the first diffraction order is formed.

The plane light monochromatic wave, normally falling onto the grating input plane, is described by the following expression:

$$e(ct - z) = E_0 \exp[i(\omega' t - k' z)], \qquad (4)$$

where t is time; z is the direction of wave propagation; E_0 is the amplitude; $k' = \omega'/c$ is the wave number (c is the speed of light).
Light oscillations in the grating output plane may be written as

$$e_1(t, x_1, z_1) = E_0 \, \mathrm{T_0 r}(x_1) \cdot \cdot (1 + m \cos \mathrm{K}_0 \, x_1) \exp[i(\omega_0 i - k' z_1)], \qquad (5)$$

where z_1 is the Cartesian coordinate of the grating output plane.
Because the grating output plane coincides with the front focal plane of the Fourier lens, then the light oscillations in the output plane of the optical system may be described by the spatial Fourier transformation [Ref. 1]

$$e_2(t, x_2, z_1) = b_0 \exp(i\omega' t) \cdot \cdot \int_{-\infty}^{\infty} e_1(t, x_1, z_1) \exp(i\omega' x_1, x_2 / cF) dx_1, \qquad (6)$$

where b_0 is a coefficient; x_2 is the Cartesian coordinate in the output plane.
Eq. (6) can be re-written in the following form:

$$e_2(t, x_2, z_2) = b_1 \exp[i(\omega' t - k' z_2) \int_{-\infty}^{\infty} \mathrm{r}(x_1) \cdot \cdot (1 + \cos \mathrm{K}_0 \, x_1) \exp(i\omega_x x_1) dx_1, \qquad (7)$$

where $b_1 = b_0 E_0 \, \mathrm{T}_0; z_2$ is Cartesian coordinate of output plane of optical system.
The result of integration of Equation. (7) is well known, Ref. 2 is an example. It contains the zero order and two first orders. The first order may be written in the form

$$f_1(t, x_2, z_0) = b_2 \frac{\sin[0{,}5(\omega_x - \mathrm{K}_0)L]}{0{,}5(\omega_x - \mathrm{K}_0)L} \cdot \cdot \exp[i(\omega' t - k' z_2)], \qquad (8)$$

$$b_2 = 0{,}5 m b_1 .$$

Eq. (7) may be written in the form

$$f_1(t, x_2, z_2) = = b_2 \frac{\sin 0{,}5 \frac{x_2}{cF}\left(\omega' - \frac{cF}{x_2}\mathrm{K}_0\right)L}{0{,}5 \frac{x_2}{cF}\left(\omega' - \frac{cF}{x_2}\mathrm{K}_0\right)L} \cdot \cdot \exp(i\omega' t) \cdot \exp(i\varphi), \qquad (9)$$

where $\varphi = k' z_2$.
Notations $\omega = \omega(x_2) = cF \, \mathrm{K}_0 / x_2$ and $T_0 = x_2 L / cF$ allow us to write function $f_1(t, x_2, z_2)$ in the form

$$f(t, x_2, z_2) = b_2 \frac{\sin[(\omega - \omega')0{,}5T_0]}{(\omega - \omega')0{,}5T_0} \cdot \cdot \exp(i\omega' t) \exp(i\varphi). \qquad (10)$$

Function $f_1(t, x_2 z_2)$ is the response of the optical network to a plane monochromatic light wave. Such wave may be treated as the result of a linear shift operator $\widehat{\mathrm{W}}$ (with dimensional scale transformation) applied to the light oscillation $e(t)$, i.e.

$$e(ct - z) = \widehat{\mathrm{W}} \, e(t). \qquad (11)$$

Then the law of transmission of light oscillations $e(t)$ by the optical network in the operator form is given as

$$f_1(t, x_2, z_2) = \widehat{\mathrm{F}}_x \widehat{\mathrm{D}}_x \widehat{\mathrm{W}}_x \exp(i\omega' t) = = \widehat{\mathrm{F}}_x \widehat{\mathrm{D}}_x \widehat{\mathrm{W}}_x \widehat{\mathrm{F}}_t^{-1} \delta(\omega - \omega'), \qquad (12)$$

where $\widehat{\mathrm{F}}_x$ is the operator of spatial Fourier transformation; $\widehat{\mathrm{D}}_x$ is the operator of spatial truncation by the aperture of the optical system; $\widehat{\mathrm{F}}_t^{-1}$ is the operator of inverse Fourier transformation in the frequency-time domain.
Therefore, the function $f_1(t, x_2 z_2)$ describes the result of transformations of the harmonic light oscillation $\exp(i\omega' t)$ into a space-time distribution of light field in the output plane of the optical network.
In the context of the theory of spectral measurements [Ref. 3], the function $f_1(t, x_2 z_2)$ is the result of transformations (12) of the $\delta-$function in frequency space ($\delta(\omega - \omega')$) into the space-time pattern in the output plane of the diffractional optical spectrum analyzer. In this case, by definition [Ref. 4], a complex-valued function $f_1(t, x_2 z_2)$ is the spectrum spread function of a spectrum analyzer, or,

more strictly, a complex spectrum spread function [Refs. 5, 6], which determines the measurement of instantaneous complex spectrum [Ref.2]. Complex spectrum spread function (10) is necessary and sufficient condition for the measurements of instantaneous complex spectrums when the oscillation being analyzed moves in respect to a fixed window (in this case this fixed window is the aperture diaphragm). Here lies a fundamental difference between this instantaneous complex spectrum and the instantaneous complex spectrum proposed by Kharkevitch [Ref. 7], where the window moves in respect to the oscillation being analyzed.

It should be noted that instantaneous complex spectrums seem to be rare in optical spectroscopy, where the power spectrum spread function [Refs. 5, 6] is commonly used.

When an instantaneous complex spectrum is measured, the value $z_)$ means the analysis time duration in Eq. (10), and value $\omega = \omega, k_( ]$ means the time spectral frequency.

In the context of linear system theory [Ref. 8], the function $E_x, i e k_(e t_( ]$ is a response to the harmonic oscillation $p[0, c\omega' i]$. Unlike electrical circuits, the output of optical network is a two-dimensional function $E_x, i e k_(e t_( ]$ in the time-space domain. In order to go from the space-time function $E_x, i e k_(e t_( ]$ to the transmission function of a linear network [Ref. 9], it is necessary to fix a point $, k_(^) e e_(^) e t_( ]$ in the output plane of the optical network. Light oscillations in this point are described by the following expression

$$
\begin{aligned}
& E, i e k_)^(e t_( ] = m_(\frac{sK(c) \,e\omega, \omega - \omega'] z_) /}{) \,e\omega, \omega - \omega'] z_)} \cdot \\
& \cdot p[0 c c, \omega' i - x' t_( ] / = E, c\omega' e i] e
\end{aligned} \qquad (12)
$$

According to the linear system theory [Ref. 9] the definition of the transmission function of linear system is the relation between the response of the network to $p[0, c\omega' i]$ and $p[0, c\omega' i]$, i.e.

$$
\begin{aligned}
& T, c\omega' e i] = \frac{E, c\omega' e i]}{p[0, c\omega' i]} = \\
& = m_(\frac{sK(c, \omega - \omega'] z_) /}{, \omega - \omega'] z_)} p[0, c\varphi] r
\end{aligned} \qquad (13)
$$

Function (13) does not dependent on time, therefore this optical network is a linear stationary system.

Eq. (13) shows that the optical network, or the diffractional spectrum analyzer is a matched filter to a signal in the form of truncated harmonic light oscillation (rectangular pulse).

If the transmission function of the grating is defined by the sum of space harmonics

$$
\begin{aligned}
& n, k_x] = n_) i, k_x], x + \sum_F d_{FL} ; 5s2_F k_x + \\
& + d_{Fb} sK2_F k_x] e
\end{aligned} \qquad (14)
$$

then the corresponding sum of harmonic light waves will interact in the output plane where the first diffraction maximum is formed. The result of this interaction depends on the complex amplitudes of light harmonics.

Transmission function of grating (14) defines another transmission function of a linear optical network, matched with another optical dynamic signal. Therefore, Eq. (14) allows us to synthesize different matched filters in the optical range.

CONCLUSION

Presented investigations are the further development of radio-optical approach in the linear system theory. Here the method of transformation of optical oscillations by a linear optical network is proposed. The diffractional optical spectrum analyzer is discussed as an example of such approach. These investigations allow us to solve some problems of optical dynamical signal processing; the problem of matched signal filtering in the optical range is among them.

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